Spraakherkenning en -synthese, lecture 4

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Today we start Module Spectrum

- Complex numbers (lecture 4)
- Fourier analysis (lecture 5)
- Convolution, correlation, filtering (lecture 6)
- Linear prediction, z-transforms, formant measurement (lecture 7)
- TEST 2 in laptop class 8

Complex numbers

- $x^2+1=0$ has no real-valued solutions x
- Assume a number i with $i^2 = -1$
- A complex number z is of the form $z = a + bi$, where a,b $\in R$
  - a is called the *real part* of z
  - b is called the *imaginary part* of z
- $|z| = \sqrt{(a^2 + b^2)}$ is the absolute value of z (this generalizes to real z)
- a-bi is the *complex conjugate* of z

Arithmetic with complex numbers

- $(a + bi) + (c + di) = (a + c) + (b + d)\, i$
- $(a + bi) \cdot (c + di) = ac + adi + bci + bd\, i^2 = (ac - bd) + (ad + bc)\, i$

Geometric interpretation of complex numbers

- $z = a + bi$
- $\theta = \arg z$ is the angle from the positive x-axis to z
- $a = |z| \cos \theta$
- $b = |z| \sin \theta$
- $\theta = \text{atan2}\,(b, a)$ lies in $(-\pi, \pi]$
- If $|z| = 1$, then z is said to lie on the *unit circle*
- On the unit circle, $z = \cos \theta + i \sin \theta$

Taylor series expansion

- $e^{i\theta} = \cos\theta + i \sin\theta$
- $e^{i\pi} + 1 = 0$
- This often makes computations easier:
- $\cos(\alpha+\beta) = ...?$
- $\cos(\alpha+\beta) + i \sin(\alpha+\beta) = \mathrm{e}^{\mathrm{i}(\alpha+\beta)} = \mathrm{e}^{\mathrm{i}\alpha}\mathrm{e}^{\mathrm{i}\beta} =$
  $= (\cos\alpha + i \sin\alpha) \cdot (\cos\beta + i \sin\beta) =$
  $= \cos\alpha \cos\beta - \sin\alpha \sin\beta + i (\cos\alpha \sin\beta + \sin\alpha \cos\beta)$
- So, $\cos(\alpha+\beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$, and
  $\sin(\alpha+\beta) = \cos\alpha \sin\beta + \sin\alpha \cos\beta s$