# SHS solution 4: complex numbers

**4.1.** The easiest way to compute a fraction of complex numbers is to multiply both the numerator and the denominator by the complex conjugate of the denominator. This is because that makes the denominator real:

$$(2i-3)(-2i-3) = 4+9-6i+6i = 13$$

Thus, $\frac{i-4}{2i-3} = \frac{(i-4)(-2i-3)}{13} = \frac{2+12-3i+8i}{13} = \frac{14}{13} + \frac{5}{13}i$.

**4.2.** The absolute value (length) of (1+$i$) is √2. The absolute value of $(1+i)^6$ is the sixth power of this, which is 8 (the third power of the square of √2).

You can see in the multiplication formula that the complex conjugate of a product is the product of the conjugates of its constituents: $(a+bi)(c+di) = (ac-bd) + (ab+bc)i$, and $(a-bi)(c-di) = (ac-bd) - (ab+bc)i$. So the conjugate of $(1+i)^6$ must be $(1-i)^6$.

**4.3.** $1+i$ has a length of $\sqrt{2}$, and an argument (angle) of $\frac{\pi}{4}$ (= 45°). The angle of $(1+i)^6$ then has to be 270°, which is along the negative *y*-axis. So $(1+i)^6 = -8i$. Picture in class of the six steps.

**4.4.** This one is easier than we thought. The angle comes at 360°, and the size must be $\left(3\sqrt{2}\right)^8 = 3^8 2^4$, so that the number is real and positive: $3^8 2^4 = 6561 \cdot 16 = 104976$.

**4.5.** The angle of −1 − *i* is $\frac{5}{4}\pi$ (or, equivalently, $-\frac{3}{4}\pi$, and its length is $\sqrt{2}$. So the length of its square root must be $\sqrt{\sqrt{2}}$, and the angle of its square root must be $-\frac{3}{8}\pi$ (by convention, the square root of a complex number is chosen to have a positive real value, in order to be compatible with the convention for real numbers, where the square root of 9 is 3, not –3). The result is $\sqrt{\sqrt{2}}\left(\cos\frac{-3}{8}\pi + i\sin\frac{-3}{8}\pi\right)$, which is approximately 0.4550898605622273 – 1.0986841134678098 *i*. One can also get at these numbers by solving the quadratic equation $-1-i = (a+bi)^2$, which gives $-\sqrt{\frac{1}{2}+\sqrt{\frac{1}{2}}}$ for the imaginary part and $\sqrt{-\frac{1}{2}+\sqrt{\frac{1}{2}}}$ for the real part (found by Jeroen S.).

**4.6.** $e^{i\varphi} = \cos\varphi + i\sin\varphi$, so that $e^{-i\varphi} = \cos\varphi + i\sin(-\varphi) = \cos\varphi - i\sin\varphi$. When we now add $e^{i\varphi}$and $e^{-i\varphi}$, the two sine terms cancel out and we are left with $2\cos\varphi$.

# Multiplication of sound signals

**4.7.** The command in the Modify menu is **Multiply**.

**4.8.** The shortest formula is

```
self * 2
```

which tells Praat to multiply every cell by 2. This is equivalent to

```
self [col] * 2
```

which tells Praat to multiply the value in each column (time point) by 2, for every row. It is also equivalent to

```
self [row, col] * 2
```

which tells Praat to multiply the value in each row (channel) and column (time point) by 2.

**4.9.** To multiply an existing selected cosine Sound with an existing sine Sound, we select the cosine Sound and do

```
self * object ["Sound sine"]
```

or

```
self * object [5]
```

if the sine Sound happens to have the ID 5 in the list (**5. Sound sine**).

**4.10.** The product of the sine and the cosine is again a sine wave, with twice the frequency of the original sine and cosine. In fact, $\sin(2\pi\ 377\ t) \cdot \cos(2\pi\ 377\ t) = \frac{1}{2}\sin(2\pi \cdot 2 \cdot 377\ t)$.