UNIVERSITY OF AMSTERDAM

Faculty of Science

Exam

Introduction Computational Science

Bachelor Informatica

Deeltoets
Date: 27 February 2018
Time: 14-16 hours

Number of pages: 3 (including front page)
Number of open questions: 25 (1 points per question)
Total number of points: 25

BEFORE YOU START

- Check if your version of the exam is complete.
- Write down **your name, student ID number**, and if applicable the **version number** on **each sheet** that you hand in. Also **number the pages**.
- Your **mobile phone** has to be switched off and be put in your coat or bag. Your **coat and bag** should be on the ground.
- **Tools allowed**: Scrap paper. Other tools are not allowed.

PRACTICAL MATTERS

- The first 30 minutes and the last 15 minutes you are not allowed to leave the room, not even to visit the toilet.
- 15 minutes before the end, you will be warned that the time to hand in is approaching.
- If applicable, fill out the evaluation form at the end of the exam.
- You are obliged to identify yourself at the request of the examiner (or his representative) with a proof of your registration and a valid ID.
- During the examination it is not permitted to visit the toilet, unless the invigilator gives permission to do so.
- The exam paper must be handed in afterwards and may **not** be taken with you.

Good luck!

Name: ______________________ Student number: ____________

Introduction Computational Science. Partial Exam

Each question gives 1 point. For a partially correct answer, one earns a fraction of a point.

Enjoy! ;)

-Lera Krzhizhanovskaya

MODELLING AND SIMULATION

1. Computational Science studies modelling and simulation, called the 3rd pillar of science. List the other two pillars of science.
2. Give at least 3 reasons for modelling and simulation.
3. List 3 limitations and "dangers" of modelling.
4. To check the correctness of our modelling & simulation results, we apply two distinct techniques. Write down the names of these two techniques and briefly explain how they are used.
5. (A) Draw the time-state plots for 3 types of models: continuous, discrete-time, discrete-event.
   (B) Give one real-life application example for each model.

FUNDAMENTALS OF CELLULAR AUTOMATA (CA)

6. Give examples of CA applications for each Wolfram Class.
7. Consider a two-dimensional cellular automaton with three possible states {0,1,2} in Moore's neighbourhood. In your answers, please do NOT use calculator, just write the exponents.
   (A) What is the size of input alphabet α?
   (B) How many transition functions (rules) are possible?
   (C) How many rules are totalistic?
8. For the Wolfram Rule 4:
   (A) Write down the Rule in a binary form.
   (B) Starting from the initial system state 00100 (with periodic boundary conditions) at time t=0, write the next two states at t=1 and t=2.
   (C) Calculate the transient length L_{trans} and cycle length L_{cycle}.
9. Characterize this Rule 4:
   (A) What Wolfram Class does it belong to? (give a Class number and a very short description)
   (B) Calculate the rule's Langton parameter, assuming the quiescent state s=0.
   (C) Fill in this table for this Rule (yes/no, and a short explanation):

reversible?	totalistic?	probabilistic?	additive?	symmetric?	ergodic?

QUANTIFY COMPLEXITY

10. How many discrete values of the Langton parameter λ are possible in a generic CA with k states and neighbourhood size N?
11. In a one-dimensional elementary cellular automaton (ECA), with k=2 and r=1, how many Rules are possible for the Langton parameter λ=3/8? Give a number, not just a formula.
12. Draw a graph of the transient length versus the Langton parameter for an elementary CA.
    Indicate the key values $\lambda_{critical}$, λ_{chaos}, and regions where we find different Wolfram Classes.

13. Calculate Shannon entropy of an ECA by counting input states (neighbourhood configurations) for a given system state with periodic boundary conditions: 1010.
14. What is the maximum possible Shannon entropy of input states of a 4-cell ECA system?

CONWAY'S GAME OF LIFE AND UNIVERSAL COMPUTER

15. In the classical Conway's Game of Life (GoL): Moore neighbourhood, only 3 rules: Birth with 3 neighbours alive, Life with 2 or 3 neighbours alive, and Death with less than 2 or more than 3 neighbours alive. Start from a line of four "1"s (1111) surrounded by zeros (empty cells in figure below) at time *t*=0, give the next two GoL states at time *t*=1 and *t*=2.

t=0:

	1	1	1	1	

t=1: - ?

t=2: - ?

16. Explain why Game of Life is a Universal Turing Machine (UTM), and which GoL structures are used as the "building blocks" of the UTM.
17. Express AND and NOT gates using only the NAND gates.
18. Draw schematically the AND gate built from the GoL glider guns. Give two examples illustrating how (X AND Y) works: one with both X and Y being true and one with both being false.

CA-BASED MODELS AND MEAN-FIELD APPROXIMATION

19. For a forest fire CA-based simulation, plot two graphs illustrating critical transition while we increase forest density (fraction of forest burnt and time to burn down).
20. In a forest fire simulation with fixed rules, does the neighbourhood type (von Neumann's or Moore's) influence the value of critical density ρ_c? If it does, which neighbourhood yields a larger ρ_c? Write ρ_c^{Moore} <??> $\rho_c^{\text{von Neumann}}$ (less than, more than, equal) and explain why, in one sentence.
21. For a 1D ECA modelling traffic flow with cars moving from right to left, write transition rule (binary).
22. Draw the fundamental diagram of traffic flow, add axis captions, indicate different flow regimes and the point of critical transition.
23. Describe formally a CA-based model of cancer growth (with the details like in the forest fire model).
24. (A) List the conditions when the mean-field approximation is valid.
    (A) Solve analytically ODE $\frac{dy}{dx} = 5y$, with initial condition $y(0) = 1$.
    (B) Find stable(!) fix points of ODE $\frac{dy}{dx} = (1 - y)y$.
25. Write down ODE equations describing the population dynamics of three of bacteria species {A,B,C}. Use constants c1, c2, etc. for model parameters. The three bacteria {A,B,C} together produce a new set of {A,B,C} in the proportion of 2:1:1. Bacteria B also clone themselves when they eat A. Bacteria A and C cancel each other (both die if they meet each other without the presence of B). Species B naturally die of old age, species A and C live forever, unless eaten or annihilated by meting each other.