Modeling and simulating traffic congestion

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For this assignment you can build upon the code that you have developed so far. If you desire you are also free to start from scratch. This assignment document is written in such a way that no existing code is assumed.

Problem statement

It is well-known that traffic flow as function of car density exhibits a phase transition. The first phase is at low density where the cars can freely flow. The second phase is at high density, where the flow stagnates suddenly and dramatically. In Japan this phase transition seems to occur at about 25 cars per km [1], see Fig. 1. This phenomenon of 'bottleneck-free' congestion on a given stretch of freeway is our **system**. In reality this is a complex phenomenon which is created by a large collection of (diverse) human drivers on a number of traffic lanes; in addition it could be raining, the road condition could be bad, etc. But we would like to gain understanding about this phenomenon through a simple-as-possible **model**. (Think of Occam's razor!) If we can find a minimal model which explains qualitatively our observed phenomenon, then we have a very strong candidate for the most important ingredient that drives the phenomenon.

Questions

1. ECA rule 184 is known as the 'traffic rule'. Implement it; use periodic boundary conditions. Explain in what way it models bottleneck-free congestion by looking at its state transition table.
2. Show the evolution of a CA of size $N = 50$ cells for 50 time steps for the 'car' densities 0.4 and 0.9. Describe briefly what you see.
3. These are two **simulated experiments**, or simulations in short, namely the experiment of letting a given density of cars drive on a stretch of road.

   Name as many advantages as you can think of for simulating these experiments as opposed to using real cars, drivers, and roads.
4. Write a function which calculates a 'car flow' value for a given initial state for the CA. We will define it as the number of 1s ('cars') that cross your system boundary on the right-hand side, per unit time. This represents a **measurement** that we can compute on a (simulated) experiment. Use a sufficiently large number of time steps, denoted T, to measure this reliably, e.g., $T \geq 1000$. Pick any $N \geq 50$.

   Plot this car flow as function of the initial density of cars, using at least 20 density values in the range [0.0, 1.0]. For each density value you should generate multiple initial states and plot average the car flow for each initial state. Let us denote the number of initial states that you sample and average over by R. Is there a clear phase transition, and if so, at which density value?

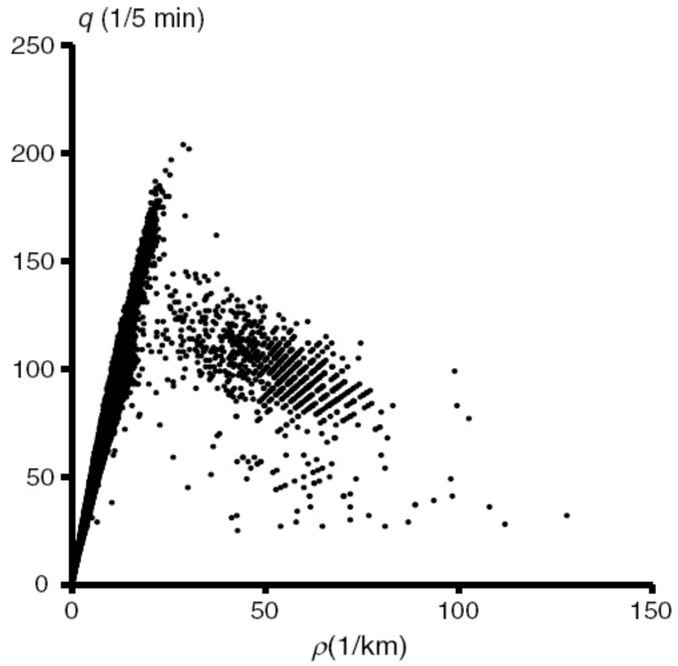

Figure 1: The typical fundamental diagram (the relation between vehicle density ρ and flow rate q) from 1 month of data measured at a point on a freeway. The critical density is nearly 25 (vehicles per km). The data were measured by the Japan Highway Public Cooperation. Copied from Reference [1].

5. Now plot the same graph but for a very low T (e.g., $T = 5$) and a very low number of initial conditions R per density value (e.g., $R = 3$). What is the effect of such 'undersampling'? Show a plot with undersampled results.

6. This effect is very important to consider both in real experiments as well as in simulations. Generally it is unknown how many cells or how many time steps are needed in order to have a reliable measurement. In real experiments it is difficult to be sure that we had a sufficient number of cars driving for a sufficient amount of time: human drivers just want to go home at some point, so you'll have to make due with the data that you gathered – undersampled or not. But in simulation we can measure this.

   Implement a function which takes the 'car flow versus density' data points[1] of exercise 4 as input and returns an automatically estimated 'position' (density value) of the phase transition as output (termed 'critical density', a scalar). (Note: the returned value does not have to be equal to one of the input densities, i.e., it may be interpolated.) Mention briefly how you implemented this.

   At which value for $T = T_{\mathbf{min}}$ do we have at least 90% probability of inferring the correct critical density? Estimate this probability by repeating your automatic detection many (at least 10) times for each T. The fraction of 'correct' values is then the probability of inferring the correct critical density. We'll say that a returned critical density value is 'correct' if it is within 0.05 of the real value. For each density value use $R = 10$ and keep it fixed. Show a plot of 'probability correct' as function of T from which it is easy to estimate

[1] In concrete terms, the input data points could be e.g. simply a list of 2-tuples (d_i, f_i), where d_i is the ith density value and f_i is the corresponding average car flow that you estimated.

$T_{\mathbf{min}}$ by visual inspection.

7. Now it is time to **analyse** the simulation results, regarding the phenomenon that we started with. Let us say that our minimal model captures the basic phenomenon very well (namely, the existence of a phase transition), using only minimal ingredients (collision avoidance). What can we conclude about the importance of other possible ingredients, such as the gender of drivers or the sizes of their cars, for explaining the existence of the phase transition? Explain why.

Assignment

Please submit to Canvas in a single archive:

1. A document in which you address each question/item above; and
2. Your code, functional and commented. Your code is required to run successfully. When run it should then output all figures which appear in your document, either on screen or in the same working directory (named e.g. as `fig1.png` or similar).

Grading

- Report document: 80%

  Is each question clearly, correctly, and adequately answered; is each plot insightful; etc.

- Python code: 20%

  Does the code work; is it well-structured and commented; does it output the same plots as in the report document; etc.

Your code will be tested for plagiarism using special-purpose software.

The grade for this assignment makes up 25% of your grade for the practical assignments for CA-part of the course (7.5% of your final grade).

Deadline: Monday, 4 March 2019, 23:59.

References

[1] Yuki Sugiyama, Minoru Fukui, Macoto Kikuchi, Katsuya Hasebe, Akihiro Nakayama, Katsuhiro Nishinari, Shin-ichi Tadaki, and Satoshi Yukawa. Traffic jams without bottlenecks—experimental evidence for the physical mechanism of the formation of a jam. *New Journal of Physics*, 10(3):033001, 2008.